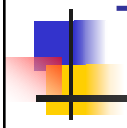

The Iman-Conover Method

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Overview

- Mechanics of IC method
- Examples of IC method
- Choices in IC method
- Software

Mechanics of Generic IC

- Inputs
  - Sample (n x r matrix) from marginal distributions
  - Correlation matrix (r x r matrix)
- Output
  - Sample re-ordered to have the same rank correlation as a reference distribution with desired linear correlation

Mechanics of IC

- Need ability to produce samples from reference multivariate distributions
  - For r lines, sample size n, desired correlation matrix S
    - Let C be Choleski root of S
      - S=C'C, where C' = transpose(C)
    - Let M be n x r matrix of independent variables where each column has mean zero and standard deviation 1
    - Corr(M) = M'M ≅ I, identity matrix (simulation error)
    - Let N = MC
    - Corr(N) = N'N = C'M'MC = C'IC = C'C = S (!)
  - Gives an easy to implement algorithm for making reference multivariate distributions

Mechanics of IC

- Fine tune reference distribution
  - If Corr(M) = M'M =: T, let T = D'D be the Choleski decomposition of T and set $E=D^{-1}$
  - Adjust M to N = MEC
  - N has exact desired correlation
    - Corr(N)= C'E'M'MEC
      = C'E'D'DEC = C'C = S
- Extra step makes IC more accurate

"Normal Copula" IC

- Apply method above with each column of matrix M equal to a random permutation (shuffling) of normal scores
  - Normal scores $\{\Phi^{-1}(p)\}$, p=1/(n+1), …, n/(n+1)
  - Scores ensure mean zero and standard deviation 1
  - Using scores reduces need to simulate normals
- Give input marginals same rank correlation as N, computed above
- Called Normal Copula IC (NCIC) because reference MV distribution is related to sample from normal copula
- NCIC is used by @Risk

Normal Copula Method

- Columns of M are random samples from N(0,1), not scores
  - Simulate random uniform u's in place of p=i/(n+1)
- Do not make adjustment to M for correlation error (no E step)
- For j=1,…,r simulate $F_j^{-1}(u_j)$ in place of matching rank order with existing sample
  - NCIC approximates percentiles of F_j with order statistics from sample from F_j

"NCIC" vs Normal Copula

- Input marginals:
  - NCIC works with existing sample from marginals
  - Normal copula generates sample from marginals by inverting distribution function
  - NCIC approximate percentiles from F_j but copula computes exactly
- Scores vs. sample from reference normals
- Only NCIC has E adjustment

"NCIC" vs Normal Copula

- Outputs
  - NCIC samples have exactly correct rank correlation matrices and approximately correct linear correlation matrices
  - Normal copula samples have approximately correct rank and linear correlation matrices

Example of IC

- SCARE Tool (see last slide)
- ComprehensiveTester.xls spreadsheet
  - Shuffles sheet – basic operation

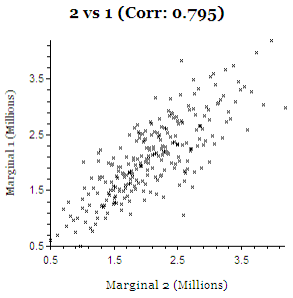
Choices in IC

- Key choice is form multivariate reference distribution; have two options
  - 1) Use Choleski-trick with different score distributions
    - Do no necessarily know distribution of resulting score marginals (e.g. sum of uniforms is not uniform)
    - Do not know form of implied copula
  - 2) Use multivariate reference sample generated in some other way
    - Many tractable alternatives
      - t-copula
      - Elliptically contoured distribution copula
      - Multivariate Laplace copula

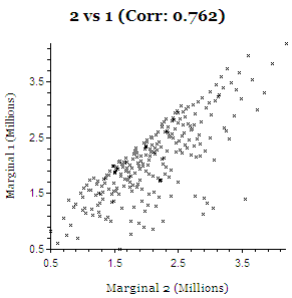
IC with Choleski-trick: varying score distribution

- Three examples with target correlation 80%, same marginal distributions
- http://www.mynl.com/MALT/ImanConover.html

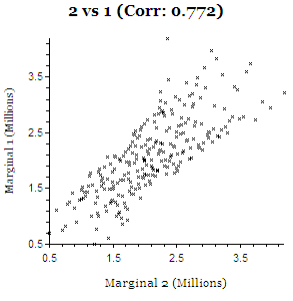
Normal Scores



Exponential Scores

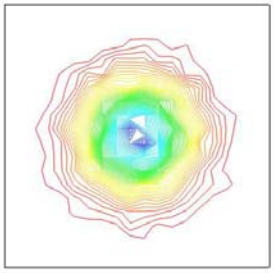


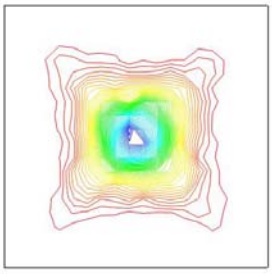
Uniform Scores



IC: other multivariate reference distribution

- Indep_Uncorr sheet
- Difference between normal and t-copulas





Software

- Working party software http://www.mynl.com/wp/default.html
  - Tools for FFTs, bivariate FFTs, Iman-Conover methods, bootstrap estimates of reserves, graphics and visualization
    - MALT – Aggregate Loss Tools, for COOS Call Program 2001
    - GREAT – Suite of actuarial tools, includes updated MALT; mostly VBA/Excel add-in
    - SCARE – Tools for Iman-Conover
  - See also http://www.mynl.com/varview/home.html
    - VarView – Visualization tools, integrates with Excel